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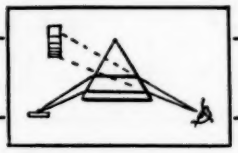

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Journal of the
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The " Depressed " Beam of the Motor Car Headlight. II

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(Received September 1, 1934)

This is an extension of the author's previous theory, originally limited to the vertical axial plane, to cover the entire reflector. The basis is established for the difference in the action of that portion of the reflector behind the focal plane from that in front of the focal plane. Computations on a representative headlamp reflector show the approximate limits of the region of anomalous behavior of the "depressed" beam.

UNDER the above title[1] the writer recently presented an analysis of the action of the vertical section through the axis of a paraboloidal reflector on light from a source displaced above the focus. The present paper extends this analysis to cover the entire reflector. The action of a reflector under these circumstances, interesting as a general optical problem, is brought into prominence just now because of the almost universal use, at least in American practice, of the two-filament bulb as a means of modifying the direction of the beam from an automobile headlamp.

In the median vertical plane the direction of a ray reflected from a given point on the parabola is affected solely by the change in the angle of incidence occasioned by switching from one filament to the other. For any other plane through the axis, the direction of a reflected ray is affected also by a rotation of the plane of incidence as the shift is made from one filament to the other. The angle through which the plane of incidence rotates has, in theory, all possible values from zero to ninety degrees. The ninety-degree value is realized, however, only in the limit as the vertex of the reflector is approached from the side. Because of interruption of the light by the bulb itself, however, the vertex of the reflector is inoperative. In actual operation, the largest value of the rotation of the plane of incidence is in the neighborhood of 55°.

Changes in direction of the projected rays accompanying a shift of filament occur in all possible planes from the horizontal to the vertical. It is principally the vertical components of these changes that interest the illumination engineer concerned with the problem of automotive headlamp design. The horizontal components are of only secondary interest. The theory presented herewith yields values of either component, but application is made only to the vertical.

Consider the action at a point of the paraboloidal reflector located at a distance ρ from the axis in a section defined by a plane through the axis making an angle θ with the vertical. Upon

[1] L. W. Taylor, J. O. S. A. **22**, 65 (1932).

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switching from the lower filament (considered to be at the focus) to the upper, the two being separated by a distance c, the total angular elevation ϵ of a ray from the point under consideration will be

$$\epsilon = \phi \cos \gamma \sin \theta + (\beta - \alpha) \cos (\theta + \phi \cos \gamma \sin \alpha), \quad (1)$$

in which, besides the notation already given, the rotation ϕ of the plane of incidence is given by

$$\cos \phi = \frac{\rho a - f \cos \theta}{[(\rho^2 \sin^2 \theta)/4 + a^2 + f^2 - 2fa \cos \theta]^{\frac{1}{2}}}, \quad (2)$$

in which

$$a = \rho(\rho^2 + 4f^2)/8fc. \quad (3)$$

The elevation ϵ will have a negative value for most portions of the reflector, though not for all. The exceptions will be discussed later. γ represents the angle between the normal and the perpendicular to the axis at the point in question and is given by

$$\cos \gamma = \rho/(\rho^2 + 4f^2)^{\frac{1}{2}}. \quad (4)$$

β represents the angle of incidence of the ray from the upper filament and is given by

$$\cos \beta = \frac{2fb - c\rho \cos \theta}{[4fb(b^2 + c^2 - 2c\rho \cos \theta)]^{\frac{1}{2}}}, \quad (5)$$

in which

$$b = (\rho^2 + 4f^2)/4f. \quad (6)$$

α represents the angle of incidence of the ray from the focus (the lower filament) and is given by

$$\tan \alpha = \rho/2f. \quad (7)$$

Eqs. (1–7) make it possible to compute the required elevation or depression of the ray reflected from any point of a given paraboloidal reflector upon switching from the lower to the upper filament. They may be derived as follows.

Expressed in cylindrical coordinates X, ρ, θ, where X coincides with the axis of the reflector and the origin is at the focus, the equation of the paraboloidal surface is

$$\rho^2 = 4f(f + X). \quad (8)$$

The length of the subnormal of a parabola is a constant, having a value half the latus rectum.

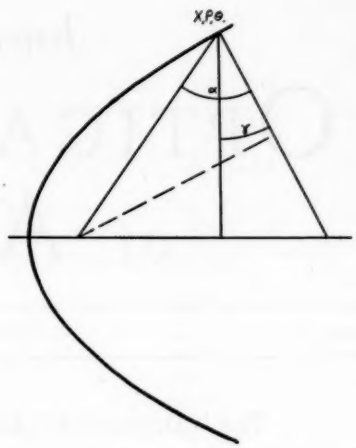

FIG. 1. Geometrical relations in plane through axis of a paraboloidal reflector.

Hence in the right triangle containing γ (Fig. 1) the side opposite γ has the value $2f$. The side adjacent to γ has the value ρ and the hypotenuse the value $(\rho^2 + 4f^2)^{\frac{1}{2}}$. Hence Eq. (4) follows directly, *viz.*,

$$\cos \gamma = \rho/(\rho^2 + 4f^2)^{\frac{1}{2}}. \quad (9)$$

For the angle of incidence α of the focal ray at the point, referring again to Fig. 1, in the large triangle which includes the angle α, the horizontal side obviously has the value $2f + x$. The other long side has the value $(x^2 + \rho^2)^{\frac{1}{2}} = [x^2 + 4f(f + x)]^{\frac{1}{2}} = 2f + x$. Hence the triangle is isosceles and the mid-point of the "base" has the coordinates $(f + x)$, $\rho/2$, θ, which, utilizing Eq. (9) may be expressed $\rho^2/4f$, $\rho/2$, θ. The length of the median is therefore

$$(\rho^4/16f^2 + \rho^2/4)^{\frac{1}{2}} = \rho(\rho^2 + 4f^2)^{\frac{1}{2}}/4f.$$

Half the length of the base is

$$(f^2 + \rho^2/4)^{\frac{1}{2}} = (\rho^2 + 4f^2)^{\frac{1}{2}}/2, \quad (10)$$

whence Eq. (7) follows, *viz.*,

$$\tan \alpha = \frac{\rho(\rho^2 + 4f^2)^{\frac{1}{2}}/4f}{(\rho^2 + 4f^2)^{\frac{1}{2}}/2} = \frac{\rho}{2f}.$$

To find the change in the magnitude of the angle of incidence at an arbitrarily chosen point on the paraboloidal reflector as the filaments are

changed, the most direct way is to find the angle of incidence β for light from the upper filament and subtract the angle of incidence α, already found, for light from the lower filament.

The directions cosines of the normal at the arbitrary point on the paraboloid are, in cartesian coordinates,

$$\lambda_1=-2f/l, \quad \mu_1=y/l, \quad \nu_1=z/l,$$

where l is the length of the normal. But

$$y=\rho \sin\theta \quad \text{and} \quad z=\rho\cos\theta. \tag{11}$$

Also, the length l of the normal is, from Eq. (10)

$$1=(\rho^2+4f^2)^{\frac{1}{2}}. \tag{12}$$

Hence the direction cosines of the normal become

$$\lambda_1=-2f/(\rho^2+4f^2)^{\frac{1}{2}}, \quad \mu_1=\rho\sin\theta/(\rho^2+4f^2)^{\frac{1}{2}},$$
$$\nu_1=\rho\cos\theta/(\rho^2+4f^2)^{\frac{1}{2}}. \tag{13}$$

The direction cosines of the line from the upper filament to the arbitrary point are, similarly in cartesian coordinates,

$$\lambda_2=x/m, \quad \mu_2=y/m, \quad \nu_2=(z-c)/m,$$

where m is the length of the line. But $m=[x^2+y^2+(z-c)^2]^{\frac{1}{2}}$. Substituting from Eq. (11),

$$m=[x^2+\rho^2\sin^2\theta+(\rho\cos\theta-c)^2]^{\frac{1}{2}}$$
$$=(x^2+\rho^2+c^2-2c\rho\cos\theta)^{\frac{1}{2}}. \tag{14}$$

Whence by substitution from Eqs. (11) and (14),

$$\left.\begin{aligned}\lambda_2&=x/(x^2+\rho^2+c^2-2c\rho\cos\theta)^{\frac{1}{2}},\\ \mu_2&=(\rho\sin\theta)/(x^2+\rho^2+c^2-2c\rho\cos\theta)^{\frac{1}{2}},\\ \nu_2&=(\rho\cos\theta)/(x^2+\rho^2+c^2-2c\rho\cos\theta)^{\frac{1}{2}}.\end{aligned}\right\} \tag{15}$$

The angle β between the lines whose direction cosines are given by Eqs. (13) and (15) is given by

$$\cos\beta=\lambda_1\lambda_2+\mu_1\mu_2+\nu_1\nu_2.$$

By substitution and a simple reduction this becomes, with the aid of Eq. (6)

$$\cos\beta=\frac{2fb-c\rho\cos\theta}{[4fb(b^2+c^2-2c\rho\cos\theta)]^{\frac{1}{2}}},$$

which is Eq. (5).

The derivation of Eq. (2) for the rotation of the plane of incidence about the normal at the arbitrary point on the reflector as the filament is shifted may be carried out most conveniently by first finding the equation of each of the planes of incidence in cartesian coordinates. For the ray from the lower filament to the arbitrary point, the plane of incidence will be defined by the origin and the normal at the point. Since the normal intersects the X axis, this is the plane which passes through the X axis making an angle θ with the vertical (the X, Z plane). Its equation is therefore

$$Y-Z\tan\theta=0. \tag{16}$$

For the ray from the upper filament to the same point, the plane of incidence will be defined by the normal and the point $(0, 0, c.)$. Since the normal intersects the X-axis at the point $X=2f+x$, this is the plane passing through the three points, $(x, y, z.)$, $(2f+x, 0, 0.)$, and $(0, 0, c.)$. Noting from Eq. (8) that $2f+x=(\rho^2+4f^2)/4f$, and using Eqs. (11), these coordinates become

$$(x, \rho\sin\theta, \rho\cos\theta),$$
$$[(\rho^2+4f^2)/4f, 0, 0.] \quad \text{and} \quad (0, 0, c.).$$

The equation of this plane is therefore

$$-X\rho\sin\theta+Y[\rho(\rho^2+4f^2)\cos\theta/4fc-2f]$$
$$-Z\rho(\rho^2+4f^2)\sin\theta/4fc$$
$$+\rho(\rho^2+4f^2)\sin\theta/4f=0. \tag{17}$$

The rotation of the plane of incidence is given by the relation

$$\cos\phi=\frac{A'A''+B'B''+C'C''}{[(A'^2+B'^2+C'^2)(A''^2+B''^2+C''^2)]^{\frac{1}{2}}},$$

where the primes and seconds of A, B and C are coefficients of X, Y and Z, respectively, in Eqs. (16) and (17). From (16), $A'^2+B'^2+C'^2=\sec^2\theta$. From (17)

$$A''^2+B''^2+C''^2=\rho^2\sin^2\theta+\rho^2(\rho^2+4f^2)^2/16f^2c^2$$
$$+4f^2-\rho(\rho^2+4f^2)\cos\theta/c.$$

Also

$$A'A''=0,$$

$$B'B''=\frac{\rho(\rho^2+4f^2)}{4cf}\cos\theta-2f,$$

$$C'C''=\frac{\rho(\rho^2+4f^2)}{4cf}\sin\theta\cdot\tan\theta.$$

Whence, by substitution and use of Eq. (3),

$$\cos\phi = \frac{\rho a - f\cos\theta}{(\rho^2\sin^2\theta/4 + a^2 + f^2 - 2fa\cos\theta)^{\frac{1}{2}}},$$

which is Eq. (2).

There remains the task of combining Eqs. (2–7) into Eq. (1) to give the vertical component of the total angular displacement of the ray reflected from the arbitrary point when the filaments are shifted. If the components of ϕ about each of the three coordinate cartesian axes be considered separately, it will be evident that each component will be proportional to the corresponding direction cosine of the normal. The component about the Y axis (the horizontal line perpendicular to the reflector axis) will be of principal interest. The component about the X axis (the axis of the reflector) will, in effect, modify the value of θ which is to be used in computing the vertical component of $(\beta-\alpha)$, the change in angle of incidence. This is, for the present application, a very minor modification. Though it has been retained throughout the present treatment, it affects numerical results seldom more than a few hundredths of a degree. The component about the Z axis (the vertical axis) produces a horizontal angular displacement which influences only the spread of the beam. It is disregarded here.

The Y direction cosine of the normal is $\cos\gamma\sin\theta$. Hence the vertical component of the angular displacement due to rotation of the plane of incidence is

$$\phi_Y = \phi\cos\gamma\sin\theta. \tag{18}$$

ϕ_Y represents a downward angular displacement of the projected ray for all portions of the reflector as the shift is made from the lower to the upper filament.

The change in the angle of incidence within the rotating plane of incidence is clearly the difference of the two angles of incidence, i.e., $(\beta-\alpha)$. In this case also, only the vertical component is of interest. This will be, as suggested above,

$$(\beta-\alpha)\cos(\theta+\phi_Y). \tag{19}$$

The total vertical angular displacement, ϵ, of the ray, upon shifting filaments, is therefore the sum of the displacements represented by Eqs. (18) and (19),

$$\epsilon = \phi\cos\gamma\sin\theta + (\beta-\alpha)\cos(\theta+\phi\cos\gamma\sin\theta),$$

which is Eq. (1).

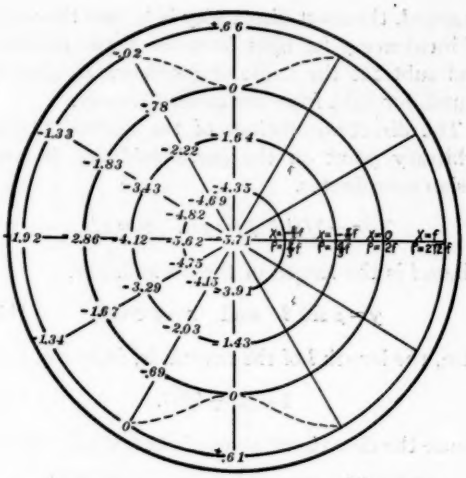

FIG. 2. Front elevation of paraboloidal reflector, showing vertical components of elevation (+) and depression (−) in various portions of the projected beam when upper or "depress" filament is in use.

For portions of the reflector substantially behind the focal plane, the two terms on the right side of Eq. (1) have the same (negative) sign, denoting downward displacement of this part of the beam upon shifting from the lower to the upper filament. But for portions of the reflector substantially in front of the focal plane, the two displacements have opposite sign, that represented by the first term on the right of Eq. (1) always remaining negative. Whether a ray projected from the forward portion of a reflector will be elevated or depressed by switching to the upper filament will therefore depend on the relative magnitudes of the two terms. These are such that the two side forward portions of a representative automobile headlamp reflector will always depress light from the upper filament, while the upper and lower forward portions will always elevate it. The amounts of these depressions and elevations, calculated for a reflector of one and one-fourth inches focal length, assuming the distance between filaments to be one-eighth inch, is shown for a number of points

TABLE I. *Computed values, for various portions of reflector, of rotation (ϕ) of plane of incidence, change (β) in angle of incidence and elevation (ϵ) of rays when change is made from lower to upper ("depress") filament.*

θ	Computed values of	$\rho/f=2\sqrt{2}$ $x/f=1$	$\rho/f=2$ $x/f=0$	$\rho/f=4/3$ $x/f=-5/9$	$\rho/f=2/3$ $x/f=-8/9$	$\rho/f=0$ $x/f=-1$
0°	ϕ	0	0	0	0	0
	β	55.39	45.00	32.05	14.08	13.32
	ϵ_1	+0.66	0	−1.64	−4.35	−5.71
	ϵ_2	+0.65	0	−1.63	−4.35	−5.71
30°	ϕ	−1.20	−2.15	−3.90	−10.55	
	β	55.31	44.98	32.36	14.88	
	ϵ	−0.02	−0.78	−2.22	−4.69	
60°	ϕ	−2.00	−3.60	−6.55	−15.90	
	β	55.06	44.94	33.07	17.35	
	ϵ	−1.33	−1.83	−3.43	−4.82	
90°	ϕ	−2.35	−4.05	−7.40	−15.88	
	β	54.76	44.92	33.90	19.12	
	ϵ	−1.92	−2.86	−4.12	−5.62	
120°	ϕ	−2.00	−3.40	−5.85	−12.23	
	β	54.44	44.95	34.57	20.97	
	ϵ	−1.34	−1.67	−3.29	−4.75	
150°	ϕ	−1.10	−2.00	−3.20	−6.60	
	β	54.21	44.98	34.99	21.98	
		0	−0.69	−2.03	−4.15	
180°	ϕ	0	0	0	0	
	β	54.12	45.00	35.12	22.34	
	ϵ_1	+0.61	0	−1.43	−3.91	
	ϵ_2	+0.61	0	−1.43	−3.91	

in Fig. 2. This represents the front elevation of a paraboloidal reflector which extends one focal length in front of the focal plane. Similar computations can of course be made for headlamps having different characteristics.

It is not feasible to derive a general expression for the locus of points on a reflector for which the sum of the two terms on the right side of Eq. (1) is zero. But the locus can be approximately identified by interpolation. It is represented by the dotted line of Fig. 2. The region within which elevation of the projected beam from an upper filament occurs will be seen to extend approximately 30° each side of the vertical at the outer edge of the reflector.

Table I gives a summary of the results of the longer portions of the computations upon which the data of Fig. 2 are based. It presents values of the rotation (ϕ) of the plane of incidence, the change (β) of angle of incidence and elevation (ϵ) of projected rays when the change is made from the upper to the lower-filament. These data may be convenient as landmarks to one who essays to make similar computations for a headlamp of somewhat different characteristics. The various points of the reflector are identified by cylindrical coordinates, but the change to cartesian coordinates may be made by use of the familiar transformation relations given in Eqs. (11) if convenience in a practical application will be promoted thereby.

The fourth and the last lines in the body of Table I, entitled ϵ_2, give for comparison the values of the angular displacement ϵ in the vertical axial plane of the reflector, computed from the previously published theory.[2] Though that is essentially a special case of the present theory, it is in a form which does not lend itself to comparison except in numerical terms.

[2] Reference 1, page 67.

On Compensating for the Aberrations of the Concave Grating

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(Received August 1, 1934)

IT is especially important to have detailed information concerning the properties of concave gratings in the vacuum region, where it is the most powerful tool for the analysis of radiation. Bowen's suggested compensation for the y^4 term by means of a $-y^2$ term is treated in detail by graphical methods, and line shapes are shown. Figures are given to show the effect of this compensation upon speed, resolving power, detectability of faint lines near strong ones, measurability of small separations, and freedom from general blackening near strong lines. Tables are given showing tolerances in the practical adjustment of concave grating spectrographs. Under certain practical conditions the tolerance in the final focusing of a spectrograph *increases* with increasing incidence angle.

§ 1.

The useful width of a concave optical diffraction grating, unlike that of a plane one, is limited by aberrations inherent in its shape: the expression for the path, object point A to grating scattering point P to image point A', contains terms in the fourth and higher powers of the grating width. Rowland[1] calculated the useful width to be limited by the boundaries (i.e., aberration $\leqslant \lambda/4$) of the first Huyghens-Fresnel zone:[2] $y=\pm(2\rho^3\lambda/(\tan\theta\sin\theta+\tan\theta'\sin\theta'))^{\frac{1}{4}}$. A calculation[3] taking the phases strictly into account, yields an optimum width some five percent greater than Rowland's approximation and shows that the resolving power is somewhat less than mN, on account of the departure of the line intensity pattern from the ideal shape $I\propto(\sin^2\alpha H)/(\alpha H)^2$.

Powers of y less than the fourth were eliminated from the aberration discussed above by confining the slit and the photographic plate to the Rowland circle (the circle $r=\rho\cos\theta$, tangent to the grating at its center). Bowen[4] has recently suggested the clever expedient of slightly displacing the slit and plate so as to compensate for the fourth power term without altering it, by introducing a second power term of opposite sign. The slit is displaced to the distance $r=(\rho+\Delta)\cos\theta$, and in order to avoid the odd-symmetric y^3 term the plate is displaced to $r'=(\rho-\Delta\sin\theta/\sin\theta')\cos\theta'$. By adjustment of the new small quantity Δ, the relative magnitudes of the y^2 and y^4 contributions to the path may be controlled at will. Bowen finds that by a judicious choice of Δ, the width $2Y$ within which the greatest aberration has a certain value, e.g., $\lambda/4$, may be increased to about $2^{\frac{1}{2}}$ times the width for $\Delta=0$.

The earlier portion (§§ 2, 3) of this paper is a detailed investigation of the consequences of using a controllable second power term to compensate for the fourth power term, bearing the same relation to Bowen's[4] work that the earlier strict investigation[3] bears to Rowland's. As the line amplitude spiral for Bowen's case is more complicated than that for Rowland's, it is relatively more in need of being supplemented by the refined method. However, the instrumental adjustments necessary to take full advantage of Bowen's refinement are so minute that in many instances they may be impractical. The later portion (§ 4) contains a method for estimating approximately the fineness of adjustment required in any case for good line quality. Detailed information concerning the properties of concave gratings in the vacuum region is more important than in any other region of the spectrum, for here there is not, and it seems unlikely that there soon will be, any more powerful tool (e.g., a "vacuum interferometer") for the analysis of fine structure. Although the theory is perfectly general, our practical remarks will be concerned particularly with the grazing incidence vacuum spectrograph.

[1] For a complete bibliography of the older work, see Kayser, Handb. d. Spectroscopie I, 450–482 (1900).

[2] The notation in this paper is that of references 3 and 4.

[3] J. E. Mack, J. R. Stehn and B. Edlén, J. O. S. A. 22, 245 (1932); 23, 184 (1933) (hereafter referred to as MSE).

[4] I. S. Bowen, J. O. S. A. 23, 313 (1933) (hereafter referred to as B).

§ 2.

Following the method of MSE, but using B Eq. (1) instead of the equation following MSE Eq. (2), we arrive at an expression analogous to MSE Eq. (8) for the intensity I in the neighborhood of a monochromatic geometric image of the slit:

$$I=\left|\int_{-H}^{H} f(\eta)e^{i(\alpha\eta-2\kappa\eta^2+\eta^4)}d\eta\right|^2,$$

where, to a sufficient (linear) approximation,

$$\kappa\equiv 2^{-\frac{1}{2}}\pi\Delta\lambda^{-1}\rho^{-2}\beta^2\sin\theta(\tan^{-1}\theta-\tan^{-1}\theta').$$

This integral may be evaluated graphically by plotting in the complex amplitude-vector plane the function:

$$w_\kappa(\alpha, H)\equiv\int_0^H e^{i(\alpha\eta-2\kappa\eta^2+\eta^4)}d\eta,$$

so that when the grating is uniformly illuminated,

$$I\propto|w_\kappa(\alpha, H)-w_\kappa(\alpha, -H)|^2.$$

Here the constant κ is proportional to the departure of the slit (or the photographic plate) from the Rowland circle, the variable α is proportional to the displacement of A' along the plate from the geometric image of the slit for the wave-length under consideration, and the variable H is proportional to the width of the grating. For a given value of κ there is a continuous family of spirals, each corresponding to a value of α, and limited in length by the value of H. The resultant amplitude of the light reaching any point A' can be found by measuring the chord ($-H$ to $+H$) of the proper spiral.

The value of κ determines the extent to which the η^4 term in the phase aberration is counteracted by the η^2 term. Of course, the second power term is predominant for small values of η, and at the line's center (the geometric image, $\alpha=0$) the central phase aberration $\phi=-2\kappa\eta^2+\eta^4$ passes through a minimum $\phi_{\min}=-\kappa^2$ at $\eta=\kappa^{\frac{1}{2}}$. The central path aberration $\epsilon=\lambda\phi/2\pi$. For the purpose of illustration in this paper we have chosen five cases. The first two:

Plane grating case: $\phi\equiv 0$, $\epsilon\equiv 0$

and

Case 0 (MSE case): $\kappa=0$, $\phi_{\min}=0$, $\epsilon_{\min}=0$

are already known in detail. The new ones are:

Case $\lambda/8$: $\kappa=(\pi/4)^{\frac{1}{2}}=0.886$, $\phi_{\min}=-\pi/4$, $\epsilon_{\min}=-\lambda/8$;
Case $\lambda/4$ (B case): $\kappa=(\pi/2)^{\frac{1}{2}}=1.253$, $\phi_{\min}=-\pi/2$, $\epsilon_{\min}=-\lambda/4$; and
Case $3\lambda/8$: $\kappa=(3\pi/4)^{\frac{1}{2}}=1.535$, $\phi_{\min}=-3\pi/4$, $\epsilon_{\min}=-3\lambda/8$.

For each new case there was plotted a family of spirals similar to MSE Fig. 2. One-half of each $\alpha=0$ spiral is reproduced here, in Fig. 2a. In all the figures shown here except Fig. 2a, the scales for similar quantities are the same, so that magnitudes are comparable between different figures.

Fig. 1 shows the relative intensity distributions on the photographic plate for grating widths corresponding to $H=1.0, 1.2, 1.4, 1.6$, in each case.[5] Each of the twenty illustrations in the figure shows a pair of lines of the same intensity, just resolved (i.e., the intensity midway between equals $8/\pi^2$ times the total intensity of either line), and their intensity sum. The percentages listed in each illustration give the distribution of the total energy belonging to a line (the left-hand one), among three regions: (1) $|\alpha H|<\pi$, within the first minima of the corresponding plane grating pattern; (2) $\pi<|\alpha H|<2\pi$, occupied by the first subordinate fringe in the plane grating pattern; and (3) $2\pi<|\alpha H|$, all the rest of the plate (the amount of energy in this outer region being represented, to scale, by the rectangle to the right).[6]

§ 3.

Let us now consider the elements that ought to enter into our choice of grating width and displacement from the Rowland circle:

(1) Strength of the central maximum of intensity—for speed, and for detecting faint lines. Fig. 2 shows the value of the central maximum of intensity for a line, as a function of

[5] It will be recalled that $H=1$ corresponds to a grating just so wide that, at the line center, the phase aberration of the light from the edge of the grating is one radian. Thus Rowland's criterion for the useful width gives $H_{\text{useful}}=(\pi/2)^{1/4}=1.120$ for Case 0, and Bowen's criterion gives $H_{\text{useful}}=(2\pi)^{1/4}=1.584$ for Case $\lambda/4$.

[6] The energies in regions (1) and (2) were obtained to an accuracy of about 1 percent, by measuring areas. (An approximate solution by series for the energy in region (3) in the plane grating case, yields 4.9 percent instead of 100–94 percent, as shown in Fig. 1.) The total energy in each case was assumed to be the same as that from the corresponding plane grating.

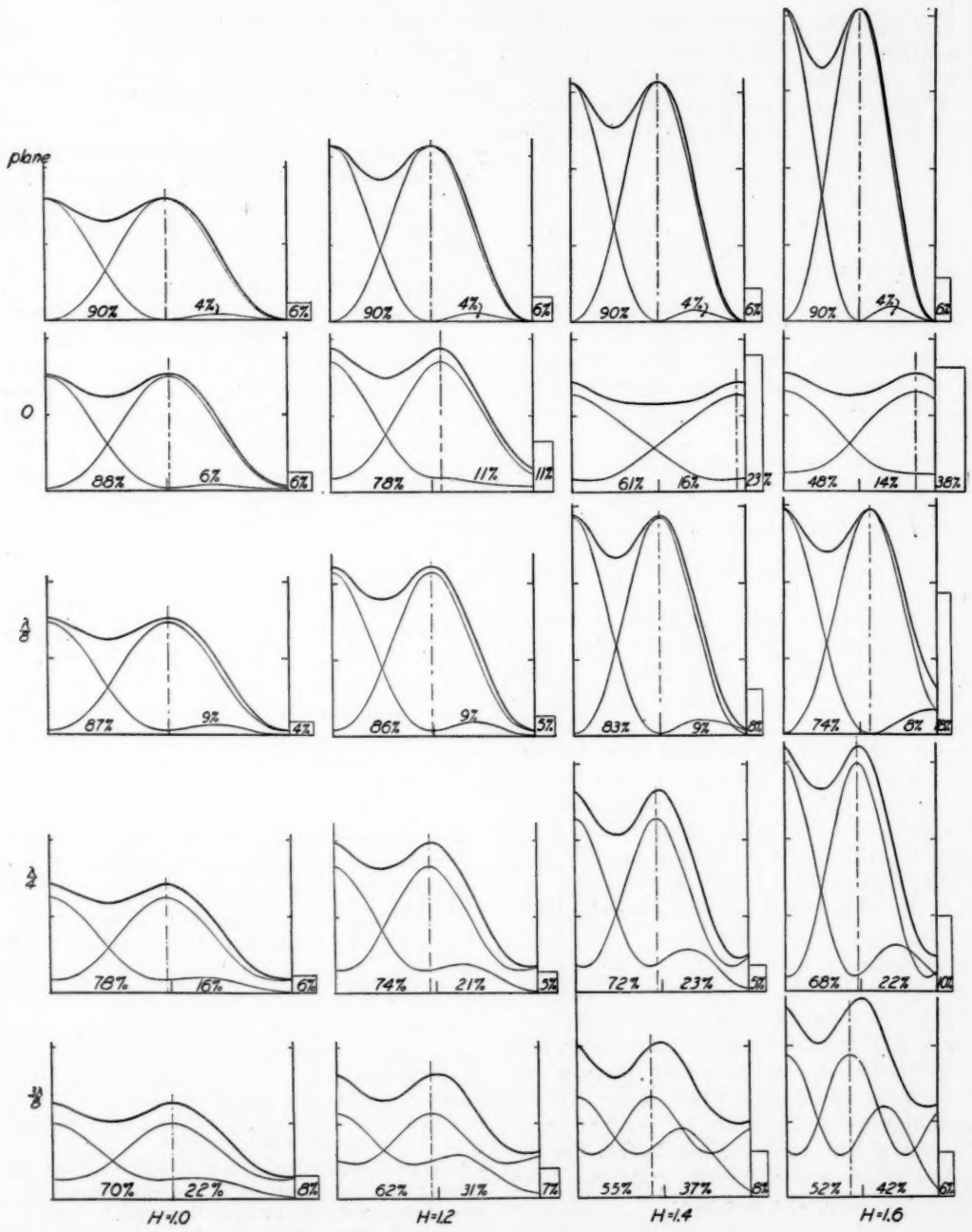

FIG. 1. Intensity as a function of displacement along the photographic plate.

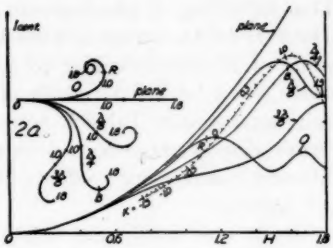

FIG. 2.

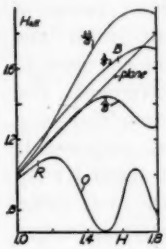

FIG. 3.

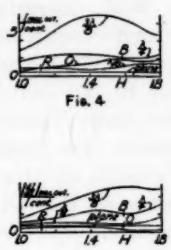

FIG. 4.

FIG. 5.

FIG. 2. Central intensity as a function of grating width. Fig. 2a. Central amplitude vector from one-half the grating.
FIG. 3. Effective grating width as a function of actual grating width.
FIG. 4. Relative intensity of the most intense subordinate maximum, as a function of grating width.
FIG. 5. Relative value of the greatest rate of change in intensity outside the central peak, as a function of grating width.

the grating width, for each of our cases. (In this and the following figures, marks R and B are placed at the H-values[5] corresponding to the limiting widths recommended by Rowland (for Case 0) and by Bowen (for Case $\lambda/4$), respectively.) The locus of greatest intensity for the various cases is shown as a dashed line; it is extended beyond Case 0 into the region where the second and fourth power contributions to the aberration have the same sign, and furnished with a scale of κ values. The figure shows a fairly sharp optimum κ of about 1.1, corresponding to ϵ_{min} about $-3\lambda/16$.

(2) Resolving power—for separating neighboring lines. Fig. 3 gives the values of H_{eff} as a function of H. It is a generalization of MSE Fig. 4. The ratio of the value of the ordinate to that of the abscissa gives the ratio of the resolving power of the grating to mN, for the case under consideration. Fig. 3 shows that the possible resolving power increases very fast with increasing κ throughout the range of cases that we have considered, even rising far above the resolving power of the corresponding plane grating! A study of the appropriate illustrations in Fig. 1 shows that as κ increases this extra resolving power is more and more accompanied by bad properties that make it undependable (see paragraphs (3) and (4) below). In no concave grating line intensity curve is the peak narrower than that of the corresponding plane grating curve. The extra resolving power comes from the contribution to the central peak of one line from the neighborhood of the first minimum of the adjacent line; this contribution is very nearly zero for the plane grating.

(3) Relative weakness of subordinate maxima—for detecting weak lines near strong ones. Fig. 4 shows the value of the strongest secondary intensity maximum, relative to the central maximum, as a function of H for the various cases we have considered. As κ is increased from zero, there is a slight fall, followed by a rapid rise, in the intensity of the strongest subordinate maximum.

(4) Flatness outside the main peak—for measurability of small separations. Fig. 5 shows the greatest rate of change of intensity outside the first minimum, relative to the central intensity. Case 0 is even better than the plane grating case in this respect, and Case $\lambda/8$ is not much worse, but the situation rapidly becomes serious with increasing κ. In Fig. 1, Case $3\lambda/8$, $H=1.6$, for instance, the steepness of the intensity curves at just the distance from the center necessary for resolution is the principal cause of the 40 percent discrepancy between the peak to peak distance on the intensity sum curve and the real center to center distance between the lines.

(5) Concentration of the area under the intensity curve, toward the center—for freedom from general blackening in the neighborhood of strong lines. The percentage distributions recorded in Fig. 1 show that for a given H, the fraction of the energy scattered outside the second minima decreases with increasing κ until it is on the order of the 5 percent found in the case of the plane grating,[6] and remains there, at least through the range of cases studied here.

Clearly, our final quantitative choice of grating width and displacement from the Rowland circle must depend upon the uses to which we intend to put our instrument. The weighting of the many factors involved is a matter of judgment. For general purposes it probably is safe enough to set κ at about 1.1 (so that ϵ_{min} is about $-3\lambda/16$) and use an H value of about 1.6; but, say, for accurately measuring the separation of a close but resolvable doublet, both H and κ ought to be smaller.[7]

It is convenient with a grazing incidence spectrograph to record in a single exposure a relatively wider region than with any other type of spectrograph, sometimes even of the order of six or seven octaves. But in a single exposure not more than one or two octaves can show the best line quality (unless the width is restricted to less than the optimum at the expense of resolving power, as in the case of near-normal incidence mountings); for, although in principle at least, both H and κ are at our disposal for any one narrow region, the relationships between H and Y and between κ and Δ depend upon λ, as well as upon θ, ρ, and m/σ, so that the choice of certain values of H and κ for one spectral region imposes upon us quite other values for other regions. Thus MSE Fig. 7, which gives the width $2Y$ for which $H=1.18$, shows a wide variation in the value of H corresponding to a certain grating width, even for a fixed angle of incidence; and quite analogously Table I, below, shows that a certain displacement of the slit produces widely different compensating effects in different spectral regions.[8]

Let us now study the relation between the displacements of A and A' from the Rowland circle and the extent of the $-y^2$ compensation for the y^4 aberration. Instead of Δ we shall calculate directly the displacement of the slit normal to the Rowland circle, which we may call Δ_n.

$$\Delta_n = \Delta \cos^2\theta$$
$$= 2^{\frac{1}{2}}\pi^{-1}\kappa\lambda\rho^2\beta^{-2}\cos\theta(1-\tan\theta\tan^{-1}\theta')^{-1}, \quad (1)$$

where 2.36β is readily obtainable from MSE Table I or MSE Fig. 7. The normal displacement of the plate holder (let us say Δ'_n) required to eliminate the y^3 term is then

$$\Delta'_n = -\Delta_n \sin\theta\cos^{-2}\theta\sin^{-1}\theta'\cos^2\theta'. \quad (2)$$

Table I gives some values of the displacements Δ_n and Δ'_n associated with a κ-value of 1.

TABLE I. Values (in cm) of the normal displacement Δ_n of the slit from the Rowland circle necessary to compensate for the y^4 aberration with a $-y^2$ term to the extent of $\kappa=1$, calculated from Eq. (1); and corresponding values of the normal displacement Δ'_n of the plate necessary to eliminate the y^3 term completely, calculated from Eq. (2), for $\rho=100$ cm. For other values of ρ, multiply by $(\rho/100\text{ cm})^{\frac{1}{2}}$. For other values of κ, multiply by κ. Plus sign (or no sign) means outward displacement from the circle. Except where θ or θ' approaches 0° (e.g., visible light, in the case $m/\sigma=24{,}000$ cm^{-1}) the functions listed in Table I and Table II vary sufficiently smoothly to allow rough interpolation.

		$m/\sigma=6000$ cm^{-1}				$m/\sigma=24{,}000$ cm^{-1}			
θ		$\lambda=10$	100	1000	5000A	10	100	1000	5000A
45°	Δn	0.00053	0.0017	0.0046	0.0058	0.00053	0.0016	0.0030	−0.026
	$\Delta' n$	.00053	.0017	.0059	.017	.00053	.0018	.0072	+ .057
80°	Δn	.00036	.0010	.0018	.0017	.00035	.00079	.00092	− .00047
	$\Delta' n$	.00038	.0014	.0094	.044	.00040	.0021	.018	+ .068
88°	Δn	.00013	.00020	.00022	.00017	.00009	.00011	.00010	− .00004
	$\Delta' n$	.00025	.0021	.022	.102	.00043	.0044	.043	+ .150

[7] Of course the MSE relationship $H_{opt}=1.18$ is not generally valid in Bowen's extension of the original Rowland problem. But after deciding, in the light of the properties discussed here, upon the best value of H to use in a given problem, one may still use MSE Table I or MSE Fig. 7 to determine the best value of the grating width, by multiplying the value given there by 1.18^{-1} times the best value of H.

[8] But it will be shown in § 4, below, that it is often possible for practical purposes to apportion the compensation in any ratio between the slit displacement and the plate displacement. We are indebted to Professor Bowen for the interesting suggestion that therefore the desired compensation may be secured for all wave-lengths simultaneously by properly figuring the plateholder. However, there appears to be no such effective way of controlling the relationship between H and the grating width.

TABLE II. Magnitude (in cm) of the normal misplacement of the slit, $|\Delta_n|$, or of the plate, $|\Delta''_n|$, bad enough to make the y^3 term in the central path aberration equal to $\lambda/4$, for $\rho = 100$ cm and $H = 1.18$. For other values of ρ, multiply by $(\rho/100\text{ cm})^{3/4}$. For other values of H, multiply by $(1.18/H)^3$.

θ		$m/\sigma = 6000$ cm^{-1}				$m/\sigma = 24{,}000$ cm^{-1}			
		$\lambda = 10$	100	1000	5000A	10	100	1000	5000A
45°	$\lvert\Delta_n\rvert$	0.13	0.23	0.33	0.44	0.13	0.23	0.32	0.48
	$\lvert\Delta''_n\rvert$	.13	.24	.42	1.3	.13	.25	.74	1.0
80°	$\lvert\Delta_n\rvert$	.027	.045	.066	0.067	.026	.040	.056	0.072
	$\lvert\Delta''_n\rvert$	.028	.062	.34	1.7	.031	.11	1.1	11.
88°	$\lvert\Delta_n\rvert$	.0034	.0045	.0073	0.010	.0029	.0042	0.0070	0.010
	$\lvert\Delta''_n\rvert$	.0066	.048	.75	6.2	.014	.17	3.2	41.

§ 4.

Heretofore we have treated our problem mathematically, as though we could make ideally accurate settings. In many cases the adjustments are quite minute, and it is important to know the practical limits of tolerance for fairly good definition. Let us retain the expression $r = (\rho + \Delta)\cos\theta$ for the point A, but introduce a new small variable Δ'' into the equation for A', thus: $r' = (\rho - \Delta\sin\theta\sin^{-1}\theta' + \Delta'')\cos\theta'$, where A and A' are initially subject only to the restriction $\Delta \ll \rho$, $\Delta'' \ll \rho$ in the plane. The process of focusing now consists in reducing the y^2 and y^3 terms to tolerable magnitudes. For any r whatsoever, there is an r' that will yield any value we choose for the coefficient of the y^2 term—i.e., in effect, any value we choose for κ. Although the y^3 term is undesirable, it is tolerable so long as its contribution to the phase difference between the amplitudes from the ends of the grating remains small compared with π. Table II gives the values of $\Delta''_n (\equiv \Delta''\cos^2\theta')$ and of Δ_n, either of which by itself would contribute $+\lambda/4$ at one edge of the grating, and $-\lambda/4$ at the other, to the y^3 term in the path aberration.[9] The values listed in Table II are approximately 20 to 200 times the corresponding displacements in Table I.

The process of focusing may consist in first adjusting the slit to within a small distance from the Rowland circle compared with the distance Δ_n listed in Table II, in order to prevent too large a y^3 term, and then adjusting the plate holder by trial to give the desired y^2 term. Fig. 2 shows that a motion of the plate equal to the Δ'_n given in Table I, or a correspondingly smaller motion of the slit, changes the maximum intensity possible by a factor of about 2, in the neighborhood of the Rowland circle. In addition to showing the displacement required to change κ by unity, the displacement listed in Table I serves as a measure of the allowable error in the final adjustment of the spectrograph. Table I shows that at grazing incidence there is generally an advantage of severalfold in tolerance, in making the final adjustments upon the plate holder rather than the slit. In order to take advantage of the differences in line shape illustrated in the figures, one ought to be able to make adjustments of at least one of the two quantities Δ_n and Δ'_n, small compared with the corresponding distance in Table I. If one cannot make such adjustments, the instrument is liable to be left, after focusing, anywhere within the gamut of κ-values.

Thus far we have neglected the finite slit width s. The greatest width that will illuminate the grating properly ($s \sim \rho\lambda/2Y$) is frequently of the order of Δ_n, Table I, particularly for the greater wave-lengths. In such a case, the shape of the line is determined by a blend from the amplitudes belonging to a continuous range of κ-values. Several papers have appeared recently[10] discussing resolving power and the effect of slit illumination upon the shapes of spectral lines. The effect of the slit-width upon the line

[9] D. L. MacAdam, J. O. S. A. **23**, 178 (1933), Table I, shows the tolerance in slit placement, but although admirably suited for mountings wherein θ and θ' stay far from 90°, the expression in terms of radial displacement measured from the grating, instead of displacement normal to the circle, is misleading when θ or θ' is near 90°.

[10] R. W. Ditchburn, Proc. Roy. Irish Acad. **39**, 58 (1930); P. H. Van Cittert, Zeits. f. Physik **65**, 547 (1930); D. C. Stockbarger and L. Burns, Phys. Rev. **37**, 920 (1931), J. O. S. A. **23**, 379 (1933).

shape was discussed in MSE as though the light falling upon the slit were generally incoherent. In the important case of the vacuum region, where the light must fall, without a condensing lens, directly upon the slit from the source, distant a few centimeters, say l, from it, the wave front falling upon the slit is almost plane: the path difference $s^2/8l$ between the central and the bounding rays introduces a phase difference on the order of $\pi\rho^2\lambda/16Y^2l$, which is ordinarily negligible. So the light falling on the slit is coherent, and amplitudes rather than intensities ought to be added in ascertaining the line shape due to a finite slit.

Pertaining to the question, what is the best angle of incidence for a grazing incidence spectrograph, Tables I and II show that, except for the very shortest wave-lengths from coarse gratings, the allowable error in the plate adjustment increases as θ approaches its limiting value of 90°. This trend is just opposite from that of the allowable error in the slit adjustment, and opposite what one might at first expect in view of the large angular displacement a small linear displacement represents, about the near-by grating as a center. This apparently anomalous behavior is understood, however, when it is realized that at grazing incidence the spectrum lines are much farther from the grating than the slit, so that $\cos\theta'$ depends much less than $\cos\theta$ upon θ; and, moreover, that Δ_n and Δ'_n contain a factor (β^{-2} in Eq. (1)) proportional to the inverse square of the useful grating width. If a grazing incidence spectrograph allows adjustment of the slit in the direction normal to the Rowland circle, small compared with the value of $|\Delta_n|$ derivable from Table II (which is of the order of magnitude of 0.1 mm) and if the final adjustment is made by moving the plate holder according to the suggestion above, the allowable error in the final adjustment increases as θ approaches 90°. Thus, whereas an 88° incidence angle allows, for most of the vacuum region, only about 0.7 as much useful grating width[7] as an 80° one (see MSE Table I), it allows about twice as accurate control of the line quality (see Table I, values of Δ'_n). We are left, then, with the argument of greater adjustability (so long as the slit adjustability satisfies the conditions set forth two paragraphs back) reenforcing those of superior dispersion and compactness and shorter optical path, in favor of $\theta\rightarrow 90°$; and only the arguments of greater usable grating width and in some cases greater slit width consistent with the best line quality (see the previous paragraph) in favor of an angle as far from grazing as the superior intensity characteristic of this type of instrument allows. It must be remembered that these arguments apply only to an ideal grating with no selective reflecting properties. Qualitative considerations lead to the conjecture that errors of ruling will usually lead to increasingly serious aberrations with increasing θ. Finally, we have but little experimental knowledge of the relative intensities of spectral lines from gratings made of various substances, as functions of the angle of incidence; further knowledge of these matters may affect our choice of θ.

Flicker Relations Within the Fovea

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OF recent years there has accumulated a considerable literature concerned with interaction effects in retinal functioning. For the most part the experiments performed have indicated the existence of a real or apparent facilitating effect of stimulation of a local retinal area on the functioning of surrounding or contiguous areas. The literature is already extensive; more than a score of papers have appeared dealing with reciprocal relations between neighboring retinal points. They leave no doubt that contiguous parts of the retina may considerably influence each other in their operation. The work of Adrian and Matthews,[1] rapidly becoming a classic in neural physiology, has adequately demonstrated the existence of a summative retinal process. The experiments of Granit and his colleagues[2] point, in the main, to a similar process in the human eye. In recent papers the writer has attempted to show that these facilitation effects are precisely describable in terms of a general mathematical function in accordance with which the heightening of sensitivity is proportional to the intensity of the inducing agent.[3] Not only are the chief phenomenal and physiological facts conclusive but the neuro-anatomical provisions have been pointed out many years ago by Cajal.[4]

However, interaction operating in the direction of mutual facilitation is not uniformly reported in the literature. Under special conditions of foveal stimulation Graham and Granit[2] report what they interpret to be evidence of an inhibitory influence at the fovea. With light patches confined to the foveal cones they found instances in which stimulation of one foveal area led to a lowering of sensitivity in other regions of the fovea. Graham has extended these experiments and has obtained data, as yet unpublished,[5] which further point to a reversal of the usual effect when stimulation is confined to the cones and when the interacting areas are unequally illuminated. These findings, if substantiated, appear highly significant. If it should be the case that an inhibitory influence is superposed upon or in some instances replaces the usual facilitatory one in the case of cone functioning there would be provided a possible basis for an explanation of color contrast as well as brightness contrast. A simple extension of the trichromatic theory of Thomas Young would accommodate the facts. In terms of this theory it would be demanded that one component, say red, inhibit the red excitatory process in surrounding parts of the retina, thus leaving, under stimulation by white light (the usual condition for the arousal of seen contrast), a balance of blue-green, i.e., a mixture of the two remaining elements. The same mechanism would do equally well for stimulation by any other color, the details working out nicely as indeed they must for any theory that is adequately framed to accommodate color-mixture facts.

A convenient test of this hypothesis would seem to be provided by a situation in which all components in a given retinal area are allowed to interact with all the elements in a contiguous area. That is to say, a critical experiment consists in stimulating one small area of the fovea with white light and measuring any change of sensitivity that may be induced in a neighboring foveal region.

A method of measuring visual sensitivity admirably adapted to the purpose at hand is that of determining critical frequencies of flicker. While there is reason to suppose that fusion frequency values are representative of purely retinal events, and to this point we shall return later, they possess the virtue of very exact deter-

[1] E. D. Adrian, and R. Matthews, *The Interaction of Retinal Neurones*, J. Physiol. **65**, 273–298 (1928).

[2] Cf. esp. C. H. Graham and R. Granit, *Inhibition, Summation, and Synchronization of Impulses in the Retina*, Amer. J. Physiol. **98**, 664–673 (1931).

[3] F. A. Geldard, *Brightness Contrast and Heymans' Law*, J. Gen. Psychol. **5**, 191–206 (1931); *Foveal Sensitivity as Influenced by Peripheral Stimulation*, J. Gen. Psychol. **7**, 185–189 (1932).

[4] Cf. R. Granit, *On Interaction Between Distant Areas in the Human Eye*, Amer. J. Physiol. **94**, 41–50 (1930) for a discussion of the essential details.

[5] Dr. Graham kindly supplied the author with an account of these experiments.

mination. If the flicker method is to be used there is considerable point in confining the interacting fields to the fovea, it having been shown by Creed and Ruch[6] that the response of the foveal region to intermittent light stimulation is quite different from that of an area only slightly removed from it. In their systematic investigation of the retina they found the flicker fusion point for a given intensity of light to drop sharply within a few degrees of the fovea. It is this very fact that makes useless for our purpose the results of such experiments as those of Lythgoe and Tansley,[7] Martin,[8] Cobb and Geissler,[9] Geldard,[10] and of similar studies in which relatively large interacting fields were used. In fact, there seem to be available no data on interacting foveal areas which might provide an answer to the question here being raised.

The experiment to be described employs a divided light field, the combined halves being confined to a one-degree subtense to insure foveal stimulation. The apparatus, schematically diagrammed in Fig. 1, consists in an arrangement for throwing two beams of light from a single source into the field, each side of which is completely independent of the other. The source is a 400-watt daylight projection lamp, the filament being supplied with a 110-volt d.c. Totally reflecting prisms, P_1 and P_2, direct the beams through the short focus lenses, L_1 and L_2, to a translucent screen, G. The screen is composed of two flashed opal glass sheets separated at the adjoining edges by a thin metal partition. The screen is shielded on the viewing side to leave visible only the divided circular field represented in the diagram. A fixation point is situated at the middle of the line separating the halves of the field. The shutter, S, controls the exposure of the entire field.

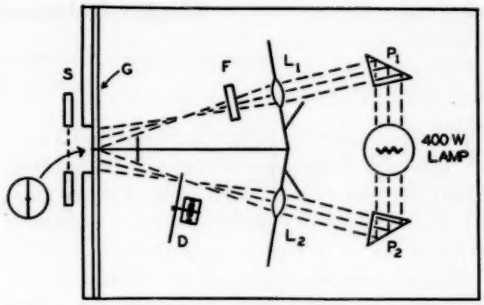

FIG. 1. Schematic diagram of the apparatus. S, pneumatic shutter; G, flashed opal glass plate; D, sector disk mounted on synchronous motor; F, neutral tint Wratten filter; L_1 and L_2, lenses; P_1 and P_2, totally reflecting prisms. The lamp is of the daylight projection type and is supplied from the d.c. mains.

Gross reductions in the intensity of the left side of the field may be effected by interposing various neutral tint Wratten filters (F) in the light path. To obtain very high intensities a mirror and lens arrangement is placed in such a position as to allow the light from the lamp to circumvent the lens mount and fall directly on the screen. The light path to the right side of the screen is cut at the focal point by a rotating disk having six open and six closed sectors. The sector disk is mounted on the shaft of a small (2 watt) synchronous motor which in turn is driven by a variable frequency oscillator designed for the purpose. This device, developed in this laboratory, has been described elsewhere.[11] The only alteration made to improve its effectiveness in the present instance was that of introducing a smooth control of the frequency changes in the oscillator. This was accomplished by the use of a movable soft iron core to vary the self-inductance of a coil. The inductance control was operated by the observer, frequency settings being made to bring about the elimination of flicker. The entire apparatus was set up in the dark room and the usual precautions were taken to shield various parts against stray light.

Throughout the main part of the experiment the right side of the field was regarded as the test area and the sensitivity of the foveal region corresponding to it was determined by measuring its fusion point of flicker. The left field, on which

[6] R. S. Creed and T. C. Ruch, *Regional Variations in Sensitivity to Flicker*, J. Physiol. **74**, 407–423 (1932).

[7] R. J. Lythgoe and K. Tansley, *The Adaptation of the Eye; Its Relation to the Critical Frequency of Flicker*, Med. Res. Council Rep. (Gt. Brit.), No. 134 (1929).

[8] L. C. Martin, *The Photometric Matching Field*, Proc. Roy. Soc. (London) **A104**, 302–315 (1923).

[9] P. W. Cobb and L. R. Geissler, *The Effect on Foveal Vision of Bright Surroundings*, Psychol. Rev. **20**, 425–447 (1913).

[10] F. A. Geldard, *The Measurement of Retinal Fatigue to Achromatic Stimulation, II*, J. Gen. Psychol. **1**, 578–590 (1928).

[11] R. C. Davis and F. A. Geldard, *An Oscillator and Synchronous Motor for Obtaining Exact Variable Speeds*, Science **73**, 369–370 (1931).

TABLE I. *Summary of results.* The brightness values are those of the contiguous (inducing) field. *N* is the number of determinations. The average critical frequency values are presented together with their standard errors.

	Observer: Gilmer		Observer: Geldard	
Brightness (ml)	*N*	Crit. Freq. (flashes/sec.)	*N*	Crit. Freq. (flashes/sec.)
—	65	51.63±.30	39	48.00±.36
3.0	27	53.07±.23	27	50.17±.27
22.6	23	53.58±.31	27	50.78±.31
109.9	35	53.90±.26	27	51.87±.36
251.5	64	55.39±.18	45	50.00±.30
494.4	42	55.17±.22	34	49.37±.26

the illumination was uninterrupted, was considered the inducing agent. Extensive data were obtained on two observers, Dr. B. von H. Gilmer and the writer.[12] After a preliminary five-minute period of dark adaptation the field was exposed, foveal fixation was immediately obtained, and the point of disappearance of flicker in the test field was secured by the observer. Exposure was continued a sufficient time for a satisfactory endpoint to be reached; usually a five-second period was adequate. Observation was in every case monocular. In successive trials various of the other five stimulating conditions were employed, the fusion point for the test field being determined in each instance. In all the experiments the test field had a mean brightness (flickering) of 96.9 millilamberts. The inducing field, when illuminated, varied in brightness between 3.0 ml and 494.4 ml, three intermediate brightnesses being used. The stimulating conditions are defined more exactly in Table I, which also serves to express the essential results.

The first column of the table states the conditions of illumination in the left half of the field, the columns labeled *N* indicate the number of determinations made under each condition of stimulation, and the average critical frequency values are given in terms of the number of flashes per second. The reliabilities of these averages are expressed as standard errors (standard deviations of the means).

The first relation that becomes apparent upon examination of the table is that for all those conditions in which the inducing field is present, whatever may be its brightness, there occurs a marked increase in the fusion frequency of the test area. Since fusion points are positively related to sensitivity it may be concluded at once that there occurs in the case of the foveal end-organs essentially the same kind of interaction process that is found when one tests the influence of extrafoveal stimulation on the functioning of the fovea or when interaction within the periphery of the retina is investigated. The presence of any contiguous light clearly makes for a heightening of sensitivity; facilitation, not inhibition, results from intrafoveal interaction.

Certain conditions surrounding the obtaining of these results require brief discussion. The values were derived from observations in which no effort was made to control the pupil. Ordinarily this is a serious source of error in quantitative visual experiments and we are here dealing with stimulation of that part of the retina which presumably has the greatest rôle to play in pupillomotor reactions. This has been pointed out long ago by Hess and his students and has been reemphasized in the recent work of Luckiesh and Moss.[13] It is to be noted, however, that had changes in pupil area appreciably entered to alter retinal illuminations the effect would have been one operating in the direction of nullifying the trend actually found to appear. To ascertain the extent to which pupil contractions might be masking the true facilitatory changes a control series was conducted in which an artificial pupil was introduced. No statistically significant differences between this and the uncontrolled conditions were found to exist. In the interest of greater ease of observation the pupillary control was dispensed with in remaining determinations.

From a consideration of the critical fusion points corresponding to the various conditions of brightness of the inducing field it becomes apparent that the degree of facilitation is not independent of intensity. The relations are more obvious in graphical form; in Fig. 2 critical frequency has been plotted against the logarithm of the intensity. As will be discovered by comparing the differences with the standard error values, all of the fusion points obtained for those situations in which the left field is illuminated are

[12] My thanks are due Mr. S. R. Wallace who kindly served as experimenter during the writer's observations.

[13] M. Luckiesh and F. K. Moss, *Area and Brightness of Stimulus Related to the Pupillary Light Reflex*, J. Opt. Soc. Am. **24**, 130–134 (1934).

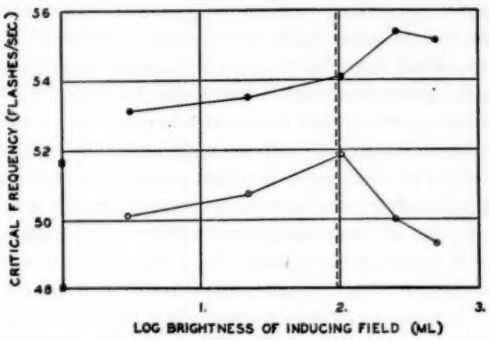

FIG. 2. The upper curve is for Dr. Gilmer's eye, the lower for the author's. The average values for the test field alone are represented by solid squares. The mean brightness of the test field is shown by the vertical dotted line.

clearly higher than the measures obtained in the absence of a facilitating field. The exact form of the function connecting the two variables is not adequately determined by these data, however. Few differences between successive points on the curves meet the test of complete statistical reliability. The general form of the curves is suggestive only. It may be of significance that the maximum facilitation appears, at least in one set of data, when the two fields are approximately equal in brightness; in this regard the results are in substantial agreement with those of Lythgoe and Tansley[7] and those of Cobb and Geissler.[9]

A matter of some interest is that concerning the persistent differences between observers. It is to be noted that the fusion points for the two observers in this experiment occupy distinctly different ranges. This phenomenon seems to be characteristic of flicker experiments. In the recent study by Hecht and Verrijp[14] a similar occurrence may be observed; for the higher intensities, at least, the values for Verrijp's eye are consistently greater than those for Hecht's. Differences of this kind were noted in the early work of Ferry[15] who directed a series of experiments towards the solution of this specific problem. He concluded (p. 205) that persistence of visual sensation is conditioned by age, that older eyes require a lower frequency to effect a disappearance of flicker. This matter needs working out in greater detail and with a sufficient number of observers to test Ferry's hypothesis.

A sufficient analysis has been made to answer the question raised earlier. It is clear that the retina does not provide the possibilities for inhibition demanded by the tentative hypothesis set up to account for contrast. But are the effects under observation in this experiment truly retinal events? Is the retina the seat of the interaction process that apparently takes place? Concerning the latter question, we are not likely to arrive at a satisfactory answer immediately, even assuming the query to be a proper one. As to the former, all the evidence points towards an affirmative reply. The best case for fusion frequencies as specific measures of retinal processes has been presented by Granit and Hammond.[16] They state: "As a matter of fact all the evidence available shows that under simple conditions the fusion frequency is determined by retinal processes alone," and cite the conclusions of the highly significant studies of Sachs[17] and Adrian and Matthews[1] in support of their contention.

If these arguments be accepted, and they seem irrefutable, it is clear that we must look beyond the retina for a mechanism suitable to effect the changes demanded by the observed facts of both brightness and color contrast. That the higher brain centers are not to be taxed with the duty is equally clear. The results of the highly ingenious but too little known experiment of Hering[18] still stand and repetition of it only convinces one that the cortex cannot be the locus of the contrast effect. Whatever may be the final outcome of our speculation concerning contrast we shall be no nearer the truth if we continue (with von Kries) to regard the phenomenon as yielding merely "accessory" facts of vision.

[14] S. Hecht and C. D. Verrijp, *The Relation Between Intensity and Critical Fusion Frequency for Different Retinal Locations*, J. Gen. Physiol. **17**, 251–268 (1933).

[15] E. S. Ferry, *Persistence of Vision*, Am. J. Sci. **44**, 192–207 (1892).

[16] R. Granit and E. L. Hammond, *The Sensation-Time Curve and the Time Course of the Fusion Frequency of Intermittent Stimulation*, Am. J. Physiol. **98**, 654–663 (1931).

[17] Reviewed by Kohlrausch, Hand. d. norm. u. path. Physiol. [12] **2**, 2, 1394.

[18] E. Hering, *Beitrag zur Lehre vom Simultankontrast*, Zeits. f. Psych. u. Physiol. der Sinn. **1**, 18–28 (1890).

Note on Higher Order Reversals in the Solarization Region

L. T. DeVore and W. J. Saylor, *Physical Laboratories, The Pennsylvania State College*

(Received August 13, 1934)

VERY little work has been done in regions of higher order reversals of the solarized latent image. In a review of the theories proposed to explain solarization, Trivelli[1] has pointed out that Lüppo-Cramer[2] discovered the probability of the existence of higher order reversals and suggested the necessity of more experimental evidence. Lubovich,[3] while photographing an eclipse of the sun, noticed that reversals occurred upon development, exhibiting two maxima and one minimum. He asserts that the effect was caused by a variation in the solar intensity as the eclipse disappeared. Since all of his exposures were of one second duration, which was too long for a normal exposure in every case, all of his images were reversed. It is the object of this paper to summarize the results of an attempt to extend the region of solarization to reversals of higher order. The experiments described here differ from that of Lubovich in that the exposures in these experiments were varied by varying the time and maintaining a constant intensity while he varied the exposures by varying the intensity and maintaining a constant time.

Procedure

Preliminary experiments were performed to determine suitable photographic emulsions, light intensities, time scales, and development time. The W. and W. Process Panchromatic emulsion was selected as the most suitable emulsion. It was necessary to employ direct sunlight in order to produce the desired solarization within a reasonable time. Both linear and logarithmic time scales were tried but the maxima and minima were crowded together to such an extent as to be almost indistinguishable when a linear scale was employed. The logarithmic time scale produced a sufficient separation to distinguish clearly the maxima and minima. This time scale also seems to lend more symmetry to the curves. An Elon Hydroquinone (Eastman type D-72) developer was used and development times of 1, 2, 4, 8, 16 and 32 minutes were tried. The 2-minute development produced the greatest contrast.

A schematic diagram of the apparatus is shown in Fig. 1. Radiation from the sun was reflected by means of a Foucault heliostat into the laboratory where it was again reflected by a plane mirror into a Hardy[4] type sensitometer. Stray light from the room was excluded from the sensitometer by means of a 12-foot tunnel with suitable baffles placed between the sensitometer disks and the plane reflecting mirror. The first 14 steps of the exposure were obtained by operating the sensitometer in the customary manner and exposing the plate during one complete cycle of the large disk in the sensitometer. The longest of the 14 exposures obtained in this fashion was 30 seconds and the time decreases in each of the preceding steps by powers of 2 giving the shortest exposure approximately 0.003 second. The next 10 steps in the exposure

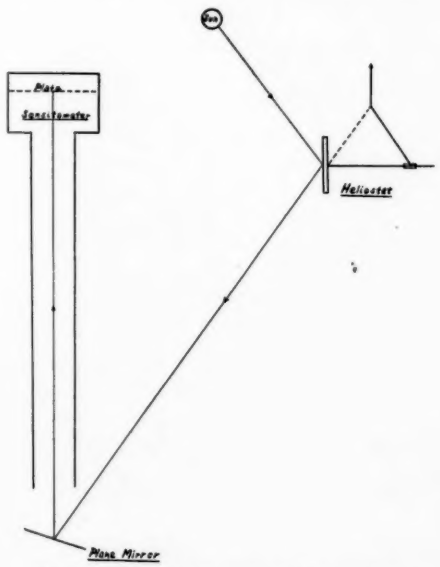

FIG. 1. Schematic diagram of the apparatus.

[1] A. P. H. Trivelli, J. Frank. Inst. **209**, 373 (1930).
[2] Lüppo-Cramer, Das Latente Bild, 37 (1911).
[3] V. Lubovich, J. O. S. A. **14**, 309 (1927).
[4] A. C. Hardy, J. O. S. A. **10**, 149 (1925).

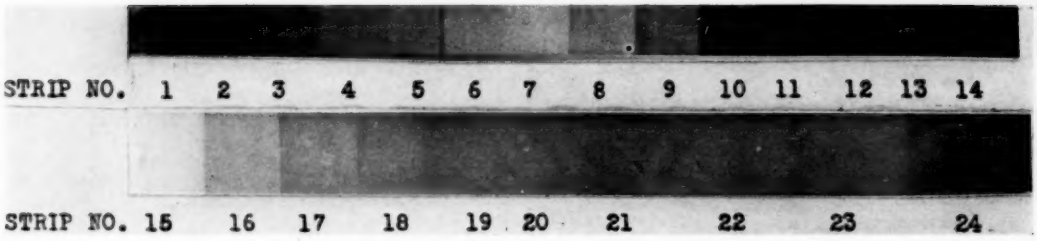

FIG. 2. Samples of sensitometer strips.

were obtained by discarding the sensitometer disks, operating the plate shutter by hand, and timing the exposures with a stopwatch. The shortest of this series of exposures was 30 seconds and the successive exposures were increased by powers of 2 giving a maximum exposure of 4 hours and 16 minutes.

DATA

Fig. 2 shows samples of the sensitometer strips obtained. The upper and lower photographs cannot be compared since in each case they were printed to give the best contrast. Actually strip number 14 of the first photograph represents exactly the same exposure and the same density as strip number 15 of the second photograph. These were identical on the negative. The maxima and minima show up clearly on the prints. It will be observed, of course, that the light sections on the print correspond to high densities on the negative and vice versa. Fig. 3 is the curve representing the density plotted against the logarithm of the time from the strips in Fig. 2. Two maxima are clearly indicated since the density is decreasing in the last step. It will be noted that there are three straight line sections in the curve and that the slope of the straight line section in the normal exposure region is exactly the same as that in the region of the second reversal of the solarization. Unfortunately insufficient data are available to afford a comparison between the slopes of the first and third reversal regions. The third reversal has just started. In general appearance, the curve seems to be a repeating process with decreasing straight line sections. The first maximum has a density of 2.21 while the second maximum has a density of 1.53. The minimum density between the first and second reversals is 1.4.

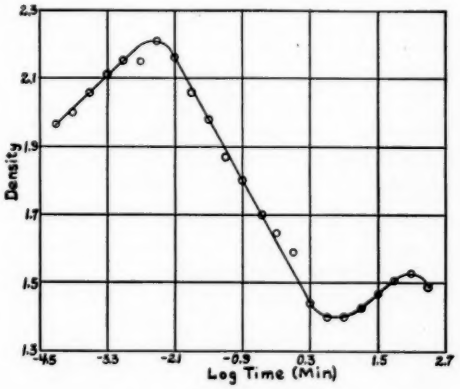

FIG. 3. Density-log time curve.

The negatives showed very interesting color effects. By transmitted light, the sectors representing short exposures demonstrated a yellowish hue which gradually changed to a bluish hue at the longer exposures. By reflected light the reverse was true.

In conclusion, the authors wish to express their appreciation for many valuable suggestions received from Dr. D. C. Duncan.

A Visual Thresholdometer

M. LUCKIESH AND FRANK K. MOSS, *Lighting Research Laboratory, General Electric Company, Nela Park, Cleveland*
(Received September 7, 1934)

THE relative visibility of different objects or the same objects under different physical conditions is usually determined by lowering their visibility until the threshold conditions are reached and expressing the visibility of the various situations in terms of one of the four variables of the visual threshold.[1] The use of the test-card of the ophthalmologist is an application of this method. In this case, the relative visibility under different conditions is expressed in terms of the threshold size of the test-object. An alternative method involves a direct measurement of three of the characteristics of the stimulus and computing the concomitant value of the fourth. The known relationships among the four variables, which have been determined empirically for single objects of simple geometric design, form a frame of reference for such computations. This method is applicable when the physical characteristics of the objects of regard are definitely describable. However, in many cases these characteristics are ambiguous. In reading, for example, entire words or even phrases are recognized during a single fixational pause of the eyes and hence the visual pattern is one of great complexity. If the visual task also involves color or highlights and shadows, the problem of analysis by the fundamental factor method may be hopelessly complicated.

In general, the first method for appraising relative visibility is the more reliable and universally applicable. Complexities in brightness, color and pattern of the objects to be seen are not necessarily a handicap in the use of this method. Its obvious disadvantages are the tediousness of the experimental technique and the not unusual difficulty encountered in attaining threshold conditions. The instrument described herein was designed to minimize these difficulties in appraising relative visibility.

The apparatus developed for this purpose is illustrated in Fig. 1. The instrument involves an arrangement whereby circular photographic films, placed directly in front of the eyes, may be rotated simultaneously by the turning of a knob. The density of these films increases uniformly from a relatively clear portion which transmits approximately 80 percent of the light, to a very dense portion which transmits less than 2 percent. Since these circular photographic gradients are placed in front of artificial pupils in an opaque screen, most of the light entering the eyes is transmitted by small definitely known areas of the films. The subject wears the device as ordinary eyeglasses are worn and slowly adjusts the controlling knob until the visual threshold of the object of regard is reached. This procedure is quite similar to that employed in operating a visual photometer. Although there are rather wide systematic differences in the results obtained for different subjects, due to variations in their criteria of threshold visibility, the individual results are quite consistent. The scale of the instrument is arbitrarily calibrated in terms of the transmission of the films.

The photographic film not only reduces the apparent brightness of the visual field, but also lowers the contrast of objects with their backgrounds due to the diffusing characteristics of the film. It is obvious from the data presented in Table I that the reduction in contrast is the more effective of these two factors in producing

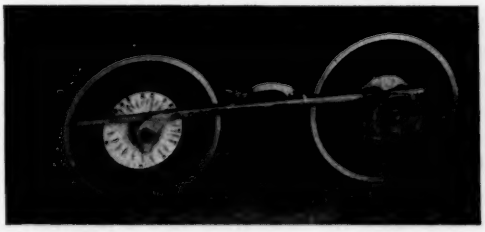

FIG. 1. A new instrument for appraising the relative difficulty of various visual tasks. It consists essentially of two gradated photographic filters which are so arranged that they may be rotated in front of the eyes thus gradually reducing the visibility of objects of regard. The users wear the instrument as ordinary eyeglasses are worn.

[1] P. W. Cobb and Frank K. Moss, *The Four Variables of the Visual Threshold*, J. Frank. Inst. 205, 831 (1928).

TABLE I. *Percent transmission of the photographic filters corresponding to the visual thresholds of various tasks. The values presented are averages of 3 series of 5 measurements each for each subject.*

Visual Task	Scale value
Reading (8 subjects)	
Bodoni Book Monotype	
6-pt.	6.2
8-pt.	5.0
10-pt.	4.1
12-pt.	3.3
14-pt.	2.5
Telephone Directory	
White pages	9.4
Yellow pages	8.3
Newspaper	
Articles 6-pt. Agate	6.6
Want Ads 5-pt. Agate	8.0
Stock Quotations 5-pt. Agate	10.0
Typing	
White paper	3.9
Yellow paper	3.6
Typing on white paper	
Original	4.4
First carbon	5.1
Sixth carbon	5.7
Typing on blueprints	5.0
Handwriting (of subjects)	
Soft pencil	5.6
Blue ink	2.3
Shorthand Notes (of subjects)	8.7
Other tasks (3 subjects)	
Discrimination of "black" thread on "black" cloth	22.0
Setting Dividers on:	
Steel Scale 1/64" Div.	7.6
Yellow Wood 1/16" Div.	5.5
Reading micrometer calipers	9.1
Setting slide rule	8.8

threshold conditions. This fact was well established heretofore.[2] It will be noted from the table of data that the transmitted light required for threshold visibility is about 50 percent greater for the 6-point book type than for the 12-point type with this particular film gradient. If brightness were the only factor involved, a change in quantity of light of approximately a hundredfold would be required to compensate for the difference in the sizes of these two type-faces.[1] Hence, the reduction in brightness may be unimportant in many practical uses of this instrument. In such cases, the calibration may be regarded as a function of contrast alone.

[2] M. Luckiesh and Frank K. Moss, *Seeing—A Partnership of Lighting and Vision*, Williams and Wilkins Co., Baltimore, 1931.

It is obvious that so-called visibility measurements are merely exact specifications of the physical characteristics of the stimulus for the threshold condition. Such data do not even involve a concept of visibility as a variable since they involve but a single point of reference. Hence the somewhat ambiguous character of the calibration of the present instrument is not a serious handicap when considered from a fundamental or absolute viewpoint. In practice, it may be used either as a transfer medium for establishing conditions of equal visibility for dissimilar visual tasks or as an indicator for appraising the relative visibility of various objects whose complexity is such as to preclude analysis by the fundamental factor method. In the latter case, it is obvious that there is no need for reference to a scale which possesses known empirical characteristics.

A variety of practical uses of this device may be cited. For example, it may be used as a means for appraising the relative effectiveness of light from sodium-vapor and tungsten-filament lamps for highway lighting under practical conditions. This application involves a determination of the visual threshold for a given test-object under a constant level of illumination provided by the sodium-vapor lamps. Similar observations could be made under two different levels of illumination from tungsten-filament lamps. These levels of illumination would be selected to provide visibility conditions which were unquestionably superior and inferior, respectively, to those resulting from the sodium-vapor light. By interpolation it would be possible to determine the intensity of illumination from tungsten-filament lamps which is equivalent to a definite level of illumination from sodium-vapor lamps.

The instrument may also be used in various practical problems which involve the improvement in conditions for seeing by the method of trial. In typography, for example, the effect of changes in size and style of type-faces or in color and surface of papers may be determined in a positive manner. It would be difficult to obtain such information by unaided subjective methods of appraisal. This is particularly true where the successive changes in factors which affect visibility are relatively small.

A few measurements taken with the "thresholdometer" are presented in Table I. With the instrument as a transfer medium, it will be noted that 8-point Bodoni book-type on white paper is equivalent in visibility with typewritten matter on blue-print paper. It also appears that the reading of shorthand notes is about as difficult as operating a slide rule, and that both of these tasks are easier than reading the white pages of the telephone directory.

The instrument offers a fairly reliable means for appraising the relative visibility of a far greater number and variety of objects than would be possible by the tedious methods usually employed in physiologic optics. Combined with recent new knowledge of seeing such devices can be helpful in substituting scientific specification of light and lighting for empirical specifications whose shortcomings are beginning to be more generally recognized. There is a great need for simple devices which appraise seeing conditions. This and other simple ones[3] are at least beginnings. As the science of seeing develops it is likely that new facts and correlations will reveal other possibilities. Unfortunately there will always remain the handicap of the inconstancy of visual and mental focus and also the variation among individuals. These are inherently involved in seeing.

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[3] M. Luckiesh and Frank K. Moss, *The Applied Science of Seeing*, Trans. I. E. S., 1933.

Annual Report of the Secretary of the Optical Society of America, Inc., for the Calendar Year 1933

Membership Record

January 1, 1933 to January 1, 1934

	Honorary	*Regular Annual Payment*	*Regular Life Payment*	*Associate Individual Annual Payment*	*Associate Corporation*	*Total*
Membership as of record in the books of the Secretary and Treasurer 1–1–33	2	412	6	242	10	672
Died in 1932 not reported until 1933		1				1–
Resigned as of 1–1–32 not previously reported				1		1–
Membership as of 1–1–33	2	411	6	241	10	670
Gains during 1933						
By election during 1933 as of 1–1–33	0	0	0	7	0	7+
By withdrawal of resignation	0	0	0	1	0	1+
By election as honorary members 10–19–33	2	0	0	0	0	2+
Losses during 1933						
By death	0	4	0	1	0	5–
By resignation as of 1–1–33	0	18	0	19	1	38–
By separation for non-payment of dues	0	1	0	6	0	7–
By election as honorary members 10–19–33	0	2	0	0	0	2–
Membership as of record 12–31–33	4	386	6	223	9	628
Gains between 12–31–33 and 1–1–34						
By election during 1933 as of 1–1–34	0	0	0	8	1	9+
By transfer 10–19–33 as of 1–1–34	0	29	0	0	0	29+
Losses between 12–31–33 and 1–1–34						
By transfer 10–19–33 as of 1–1–34	0	0	0	29	0	29–
By resignation during 1933 as of 12–31–33	0	3	0	3	0	6–
Membership as of record 1–1–34	4	412	6	199	10	631

Died in 1932 not reported until 1933

Ernest Howe (57 R)

Resigned as of January 1, 1932 (not previously reported)

M. E. Graber (871 A)

Elected during 1933 as of January 1, 1933

Ellsworth M. Brockway (937 A)
Lyman Chalkley, Jr. (935 A)
Martin Grabau (940 A)
Walter C. Granville (944 A)
Jean Maurice Riviere (936 A)
George L. Royer (938 A)
Murray R. Scott (939 R)

Withdrew Resignation (Resigned 12–31–32)

E. L. Harrington (768 A)

Elected as Honorary Members October 19, 1933

Edward Bausch (2 H)
Charles Fabry (88 H)

Elected during 1933 as of January 1, 1934

Carl E. Bahn (942 A)
Julian M. Blair (948 A)
Elizabeth Cohen (945 A)
Gustave Fassin (943 A)
David L. Gamble (950 A)
Richard S. Hunter (947 A)
International Printing Ink Corp. (946 AC)
Jakob Kunz (949 A)
Julian H. Webb (941 R)

Elected to Regular Membership by Action of the Board of Directors October 19, 1933, Effective January 1, 1934

Walter Edward Albertson (642 R)
John G. Albright (726 R)
Paul L. Bayley (619 R)
Edward L. Bowles (450 R)
Barnett Cohen (690 R)
Arthur F. Dittmer (623 R)
Charles G. Ferrari (839 R)
A. P. M. Fleming (569 R)
John W. Forrest (900 R)
Carl E. Foss (878 R)
G. E. Grantham (774 R)
Frederick R. Hirsh, Jr. (747 R)
Donald Hubbard (615 R)
Serge A. Korff (823 R)
Overton Luhr (921 R)
David W. Mann (743 R)
J. Franklin Meyer (596 R)
Parry H. Moon (738 R)
Donald R. Morey (889 R)
Dorothy Nickerson (545 R)
Fred H. Perrin (406 R)
Orrin Weston Pineo (566 R)
Joseph Razek (704 R)
Murray R. Scott (939 R)
Francis W. Sears (919 R)
Leland B. Snoddy (859 R)
Joseph W. Tanch (671 R)
Mary Elizabeth Warga (893 R)
Julian H. Webb (941 R)

Deaths during 1933

Paul Anderson (131 R)
Clement D. Child (820 R)
Frederick L. Gates (789 R)
Alexander S. Galajikian (740 A)
A. F. Wagner (35 R)

Resignations during 1933 as of January 1, 1933

William D. Appel (599 R)
Erich Bandoly (257 A)
Harold R. Barnes (663 A)
James Barnes (894 R)
Frederick Bedell (483 R)
Leland J. Boardman (481 A)
Theodore W. Case (215 R)
R. W. Cheshire (315 R)
Herbert A. Clark (121 R)
Alexander L. M. Dingee (725 A)
William L. Dolley, Jr. (699 R)
Richard Evans (887 A)
Electrical Testing Labs. (458 AC)
Maurice Ewing (791 A)
H. E. Farnsworth (840 R)
Kenneth Hartley (777A)
R. M. Holmes (807 A)
A. L. Johnsrud (737 R)
Richard N. Jones (874 A)
Robert James Lang (541 R)
Charles J. Ling (381 A)
S. Nakamura (120 R)
George H. Needham (631 A)
A. D. Power (744 A)
H. K. Richardson (854 A)
E. L. Robinson (610 A)
John R. Roebuck (448 R)
P. A. Ross (515 R)
George R. Sears (718 A)
Raymond S. Seward (797 A)
Alpheus W. Smith (313 R)
Orrin H. Smith (786 A)
T. Townsend Smith (86 R)
Otto Stuhlman, Jr. (342 R)
Earle H. Warner (526 A)
M. R. Wehr (721 A)
John C. Wollensak (356 R)
Anthony Zeleny (307 R)

Resignations during 1933 as of December 31, 1933

Hollis S. Baird (929 A)
Hober D. Curtis (108 R)
George S. Haslam (574 R)
Ernest Petry (897 A)
Wayne T. Sproull (632 A)
Henry Theodore Wensel (657 R)

(*Signed*) L. B. TUCKERMAN, *Secretary*

Optical Society of America, Inc.

STATEMENT OF CASH RECEIPTS AND DISBURSEMENTS FOR THE PERIOD FROM JANUARY 1, 1933 TO DECEMBER 31, 1933

Receipts

Dues	$2,066.95	
Dues in advance	1,537.47	
Unbudgeted income	2.27	
Interest on bonds	832.48	
Interest on Rochester Trust & Safe Deposit	133.89	
Interest on Chem. Bank & Trust	.36	
Ives Medal account	21.25	
Contributions	1,000.00	
		$5,594.67

Disbursements

President's expense	$ 3.76	
Secretary's expense	857.23	
Secretary's travel expense	61.60	
Treasurer's expense	84.10	
Journal publication expense	4,722.00	
Contingencies	196.22	
Ives Medal account	17.31	
		$5,942.22
Cash Decrease Jan. 1 to Dec. 31, 1933		$ 347.55
Cash in Rochester Trust & Safe Deposit, Jan. 1, 1933	$5,973.99	
Cash in Chem. Bank & Trust, Jan. 1, 1933	1,623.31	
		7,597.30
Cash in Bank Dec. 31, 1933		$7,249.75

Bond Investments: December 31, 1933

$7,000 Atchison Topeka & Santa Fe Gen 4's 1995	$ 6,916.25
500 Fourth Liberty 4¼'s 1933–1938 (Ives Fund)	500.00
9,500 Fourth Liberty 4¼'s 1933–1938	9,719.64
3,600 U. S. A. Treasury 4¼—3¼'s	3,565.87
	$20,701.76

I certify that the above report is correct to the best of my knowledge and belief

(*Signed*) HENRY F. KURTZ, *Treasurer*

February 15, 1934

The undersigned, having been duly appointed to audit the books of the treasurer of the Optical Society of America from January 1, 1933 to December 31, 1933, hereby certify that the treasurer's official report of receipts, disbursements and investments is correct as shown by records, bank statements, vouchers, etc.

(*Signed*) CARL L. BAUSCH
C. W. FREDERICK

Treasurer's Report to Board of Directors Optical Society of America for 1933

The audited accounts show receipts of $5,594.67 and disbursements of $5,942.22 which indicates a loss of $347.55 for the year's operations as against a budgeted loss of $175.00 for the year and against an estimated loss of $46.31 made by the treasurer at the time of the October, 1933 meeting. Below is an adjustment of these items for the purpose of eliminating disbursements chargeable to the activities of 1932 and receipts to be credited to 1934 operations:

Audited receipts Dec. 31, 1933		$5,594.67	
Less 1934 dues paid in advance during 1933	$1,537.47		
Contribution A. Lomb Estate	500.00	2,037.47	
		$3,557.20	
Plus 1933 dues paid in advance during 1932		1,772.46	
Net operating receipts for 1933		$5,329.66	$5,329.66
Audited disbursements Dec. 31, 1933		$5,942.22	
Less Printing chargeable to 1932	$ 153.25		
Publication expense paid in 1933 chargeable to 1932	1,122.00	1,275.25	
		$4,666.97	
Plus publication expense paid in 1934 chargeable to 1933		120.11	
Net operating disbursements for 1934		$4,787.08	$4,787.08
Net profit on 1933's operation			$ 542.58

The above shows that the net operations for the year 1933 provide us with a profit of $542.58 instead of the budgeted loss of $175.00 or the loss of $46.31 estimated in October, 1933. It is true that this amount includes an unbudgeted amount of $500 contributed by the Eastman Kodak Company, but your treasurer is gratified to report that the finally balanced operations show a profit of $42.58 as against the carefully considered prospects of last October which indicated at best a loss of $46.31, without consideration of this generous contribution.

(*Signed*) HENRY F. KURTZ, *Treasurer*